Design Procedure for Inhomogeneous Coupled Line Sections

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Abstract —**This paper presents design formulas and a procedure for the design of inhomogeneous coupled line sections as an approximation to a series open circuited stub for application in the realization of microwave pseudo high-pass filters. The accuracy of the design equations is evaluated through the design and test of a seventh-order filter and it is found that the formulation predicts the performance of the section well beyond the quarter-wave frequency.**

I. Introduction

Wide-band, high-selectivity pseudo high-pass filters in suspended stripline have become extremely popular due to their excellent performance and very small size [1], [2]. These filters all make use of series open circuit stubs, and in the design they are approximated by short, overlay-coupled line sections, as shown in Fig. 1. The extent to which the filter performance tracks the design value is obviously a function of the correctness of this approximation.

In this paper, new design equations are derived that model the coupled line section almost exactly up to the quarter-wave frequency. The effect of the fringing capacitance from the open ends of the coupled lines is also included in the design.

A trial seventh-order filter was designed and tested. The filter gave excellent performance with the passband extending from 2 GHz to 9.5 GHz.

II. Derivation of the Design Equations

The basis of the approximation to be used in this paper is the matching of the *ABCD* parameters of the two sections at two separate frequencies. Zysman and Johnson [3] give the *ABCD* parameters of an inhomogeneous coupled line section as

$$A = D = \frac{Z_{0e}\cot\theta_e + Z_{0o}\cot\theta_o}{Z_{0e}\csc\theta_e + Z_{0o}\csc\theta_o} \tag{1}$$

$$B = j\frac{Z_{0e}^2 + Z_{0o}^2 - 2Z_{0e}Z_{0o}(\cot\theta_e\cot\theta_o + \csc\theta_e\csc\theta_o)}{2(Z_{0e}\csc\theta_e - Z_{0o}\csc\theta_o)} \tag{2}$$

$$C = \frac{2j}{Z_{0e}\csc\theta_e + Z_{0o}\csc\theta_o} \tag{3}$$

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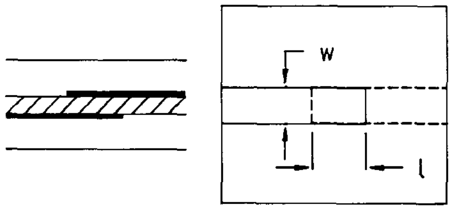

Fig. 1. Overlay-coupled line section.

where

$$\theta_e = \frac{2\pi f l}{c\sqrt{\epsilon_{\mathrm{eff}\,e}}} \tag{4}$$

$$\theta_o = \frac{2\pi f l}{c\sqrt{\epsilon_{\mathrm{eff}\,o}}} \tag{5}$$

The *ABCD* parameters of a series open circuited stub are given by

$$A = D = 1 \tag{6}$$

$$B = \frac{-jZ_c}{\tan\theta} \tag{7}$$

$$C = 0. \tag{8}$$

Fig. 2 compares the variation of the *ABCD* parameters versus frequency for the stub and the coupled line section. The parameters $A = D$, B, and C are compared separately in Fig. 2 in parts (a), (b), and (c), respectively. It is obvious from the graphs that the latter can be made to approximate the stub quite closely up to the quarter-wave resonant frequency of the stub.

As $A = D$, there are in effect three parameters to match. In a typical cross section, for the coupled lines, the line width is usually much larger than the spacing between the two lines, as shown in Fig. 1. Consequently, the even-mode impedance of the pair of coupled lines is very much larger than the odd-mode impedance, $b \gg s$, $Z_{0e} \gg Z_{0o}$. If, furthermore, the coupled line section is made much shorter than the corresponding series stub, from (1) to (5) we find that $A = D \simeq 1$ and $C \simeq 0$.

The remaining parameter is matched for the two elements at two frequencies, namely the cutoff frequency, f_c, of the high-pass filter to be realized and the resonant frequency of the stub to be approximated, f_0. Only two frequencies can be used, because that is the number of orders of freedom available in the expressions for B. These

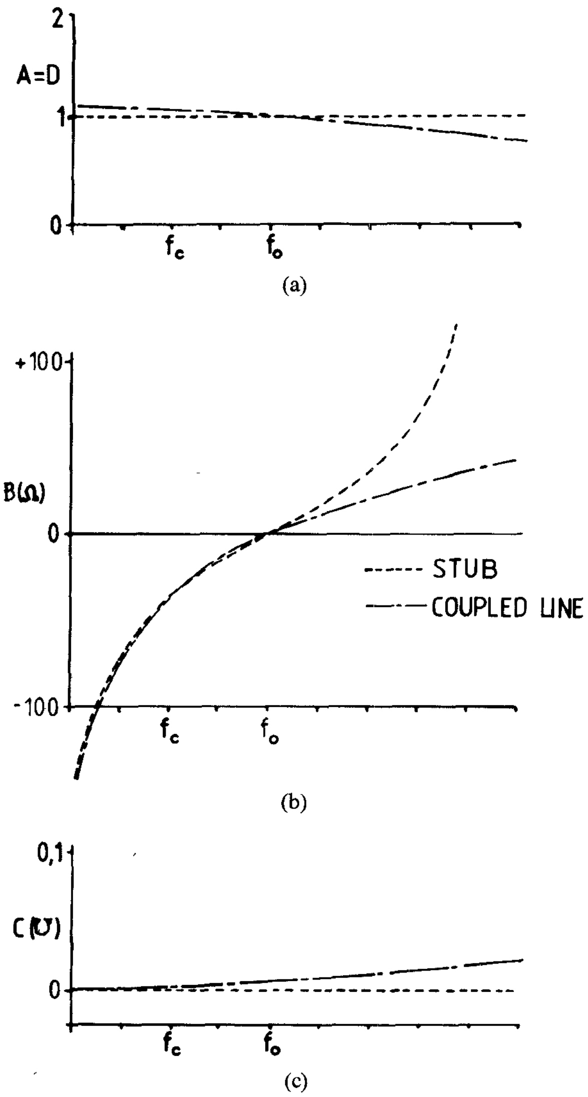

Fig. 2. Variation of the *ABCD* parameters of the coupled line section and the open series stub with frequency. (a) $A = D$. (b) B. (c) C.

two frequencies can be chosen arbitrarily; however, the two sections will be identical where the choice is made.

Choosing the band edge of the filter ensures that there will be no bandwidth errors in the filter design. The choice of the quarter-wave frequency of the stub, however, is somewhat arbitrary, although very close to optimal. This is because making the two responses intersect at the inflection point of the stub response ensures the same type of slope over the whole of the first 90° of the parameter responses.

Finally, we find that

$$B = 0 \qquad \text{at } f_0$$

$$B = \frac{-jZ_c}{\tan\theta_c} \qquad \text{at } f_c.$$

For the calculation of the even- and odd-mode impedances, we first approximate the inhomogeneously filled cross section by a uniformly air-filled cross section, as shown in Fig. 3. Under the circumstances of $Z_{0e} \gg Z_{0o}$, this is a good approximation, and enables the equations of

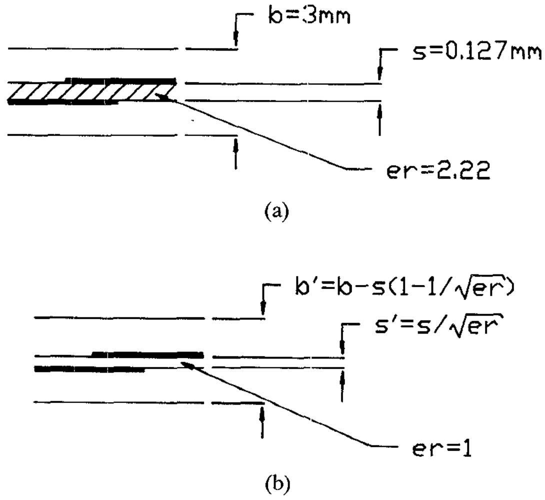

Fig. 3. (a) Cross section of coupled suspended striplines. (b) Equivalent homogeneous coupled line section.

Cohn [4] for lines in air to be used. For the equivalent cross section, the even- and odd-mode impedances are given by

$$Z_{0e} = \frac{\eta}{2\left[\dfrac{w/b'}{1-s'/b'} + C_{fe}/\epsilon\right]} \tag{9}$$

$$Z_{0o} = \frac{\eta}{2\left[\dfrac{w/b'}{1-s'/b'} + w'/s' + C_{fo}/\epsilon\right]} \tag{10}$$

where

$$C_{fe}/\epsilon = 0.4413 + \frac{1}{\pi}\left[\ln\left[\frac{1}{1-s'/b'}\right] + \frac{s'/b'}{1-s'/b'}\ln\frac{b'}{s'}\right] \tag{11}$$

$$C_{fo}/\epsilon = \frac{b'/s'}{\pi}\left[\ln\left[\frac{1}{1-s'/b'}\right] + \frac{s'/b'}{1-s'/b'}\ln\frac{b'}{s'}\right] \tag{12}$$

$$\epsilon_{\text{eff}\,e} = 1 \tag{13}$$

$$\epsilon_{\text{eff}\,o} = \epsilon_r. \tag{14}$$

If more accurate values for the odd-mode impedance and the effective dielectric constant are needed, formulas for covered microstrip, as given, for instance, by March [5], can be used.

III. Design Equations Without Inclusion of End Effect

For any transmission line that is abruptly terminated, especially in an open circuit, the resonant length is a function of the stray capacitance terminating the line. As a first-order approximation, we neglect this contribution, and under the condition $l \ll \lambda/4$, we make the approximation

$$\tan\theta = \sin\theta = \theta. \tag{15}$$

At $f = f_0$, $B = 0$. Substituting (15), (4), and (5) into (2) and solving for l yields the expression

$$l = \sqrt{\frac{c^2 Z_{0e} Z_{0o}}{\pi^2 f_0^2 \sqrt{\epsilon_{\text{eff}\,e}\epsilon_{\text{eff}\,o}}\left(Z_{0e}^2 + Z_{0o}^2\right)}}. \tag{16}$$

At $f = f_c$, (7) gives another expression for B. Once again using the approximation (15) in (2) gives

$$\frac{-jZ_c}{\tan\theta_c} = \frac{1}{2} j \frac{\theta_e\theta_o\left(Z_{0e}^2 + Z_{0o}^2\right) - 4Z_{0e}Z_{0o}}{\theta_o Z_{0e} - \theta_e Z_{0o}}. \tag{17}$$

Substituting (16) into (4) and (5), and the resulting equations into (17), we obtain, using the approximation $Z_{0e} \gg Z_{0o}$,

$$Z_{0e}Z_{0o} = \frac{Z_c^2 (f_c/f_0)^2 \sqrt{\epsilon_r}}{\tan^2\theta_c\left[1-(f_c/f_0)^2\right]^2} \tag{18}$$

and if we substitute (9) and (10) into (18), we find

$$w/b' = \frac{-y + \sqrt{y^2 - 4xz}}{2x} \tag{19}$$

where

$$x = \frac{1}{(s'/b')[1-(s'/b')]^2} \tag{20}$$

$$y = \frac{(s'/b')(C_{fo}/\epsilon) + (C_{fe}/\epsilon)}{(s'/b')[1-(s'/b')]} \tag{21}$$

$$z = (C_{fe}/\epsilon)(C_{fo}/\epsilon) - \frac{\eta^2 \tan^2\theta_c\left[1-(f_c/f_0)^2\right]^2}{4Z_c^2 (f_c/f_0)^2 \sqrt{\epsilon_r}}. \tag{22}$$

IV. Design Equations Including End Effect

If the open circuit end effect is included, it becomes impossible to obtain expressions for the line widths and lengths of the coupled line sections in closed form. At the same time, the fringing capacitances that terminate the line are different for the odd and even modes, so that there is no simple method of compensating for the end effect. The procedure followed here is to include the effects of the fringing capacitance in the effective dielectric constant for the two modes, thereby effectively changing the resonant lengths in the two modes.

The effective length of the line in each mode is increased by a length Δl such that the total capacitance of the new, compensated length of line is the same as the original line with end effect. Thus,

$$\Delta l_e C_{0e}/\epsilon = w C_{fe}/\epsilon \tag{23}$$

$$\Delta l_o C_{0o}/\epsilon = w C_{fo}/\epsilon. \tag{24}$$

The effective dielectric constants for the two modes are now given by, for $i \in \{e, o\}$,

$$\epsilon'_{\text{eff}\,i} = \left[\frac{l\sqrt{\epsilon_{\text{eff}\,i}} + \Delta l_i}{l}\right]^2 \tag{25}$$

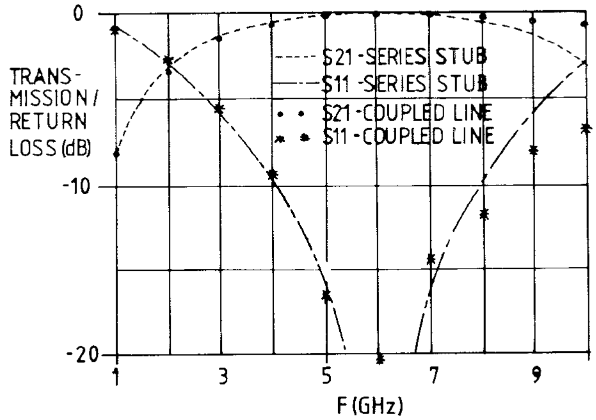

Fig. 4. Theoretical and measured return loss and transmission loss versus frequency for a coupled line section.

Substituting $C_{0i} = \eta/Z_{0i}$, we find

$$\epsilon'_{\text{eff}\,i} = \left[\sqrt{\epsilon_{\text{eff}\,i}} + \frac{w}{l}\frac{C_{fi}/\epsilon}{\eta} Z_{0i}\right]^2. \tag{26}$$

The value for $\epsilon_{\text{eff}\,o}$ can once again be calculated from [5], while $\epsilon_{\text{eff}\,e} = 1$.

We now obtain the values for l and w by solving numerically the equations for the parameter B at the two matching frequencies, f_c and f_0, using (4), (5), and (9)–(12).

At $f = f_0$:

$$Z_{0e}^2 + Z_{0o}^2 - 2Z_{0e}Z_{0o}(\cot\theta_e \cot\theta_o + \csc\theta_e \csc\theta_o) = 0. \tag{27}$$

At $f = f_c$:

$$\frac{Z_{0e}^2 + Z_{0o}^2 - 2Z_{0e}Z_{0o}(\cot\theta_e \cot\theta_o + \csc\theta_e \csc\theta_o)}{2(Z_{0e}\csc\theta_e - Z_{0o}\csc\theta_o)} = \frac{-Z_c}{\tan\theta}. \tag{28}$$

The parameter C should ideally be zero; unfortunately this is not the case, and this would limit the useful range to substantially below the quarter-wave resonant frequency of the filter.

This problem can be overcome by forcing the reflection coefficient of the element to zero as described below. The reflection coefficient can be rewritten in terms of the chain parameters as

$$S_{11} = \frac{B - Z_0^2 C}{2AZ_0 + B + Z_0^2 C}. \tag{29}$$

If we now set (27) to zero at a frequency slightly lower than f_0, we can ensure that at f_0, $B = Z_0^2 C$, and therefore ensure that $S_{11} = 0$, as is the case for the series open circuit stub. This choice unfortunately results in an uneven passband VSWR performance when applied to filters; consequently, the frequency at which $B = 0$ is chosen slightly higher than the frequency at which $S_{11} = 0$ at $f = f_0$ to ensure an even VSWR across the passband of the filter.

V. Experimental Verification

In order to verify the design procedure described above, a number of coupled line sections were manufactured and measured. Fig. 4 shows the insertion loss response of a

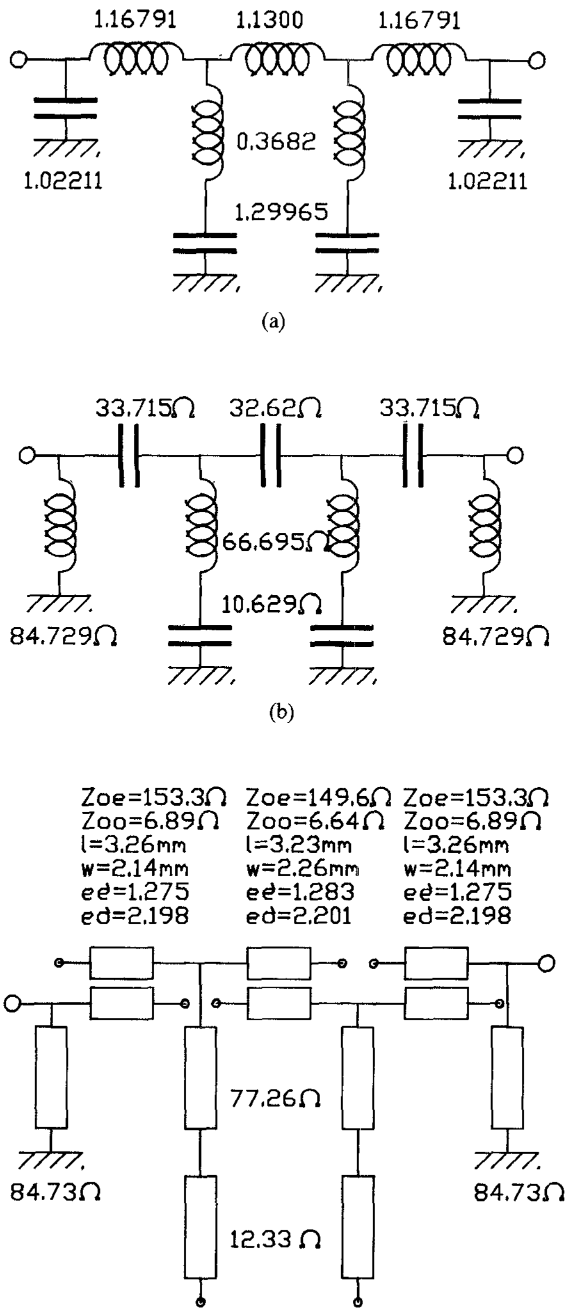

Fig. 5. Design of a seventh-order filter. (a) Low-pass prototype. (b) Prototype after transformation to high-pass and scaling. (c) Final element values.

typical section, of $Z_0 = 60\ \Omega$, with a cutoff frequency of $f_c = 2$ GHz, and center frequency of $f_0 = 6$ GHz. The substrate used was RT Duroid of 0.005 inch thickness, and dielectric constant $\epsilon_r = 2.22$. Excellent agreement between the theoretical prediction and the calculated values is noted.

VI. Trial Filter Design

A filter was designed to verify the application of the design procedure to filter applications. The filter was a seventh-order generalized Chebyshev prototype [6] with $A_m \geqslant 50$ dB, return loss more than 20 dB, $f_0 = 6$ GHz, and

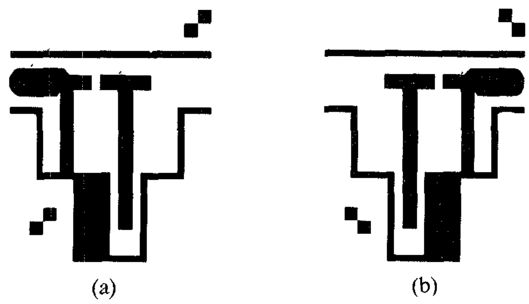

Fig. 6. Artwork of the filter before etching. (a) Upper masks. (b) Lower masks.

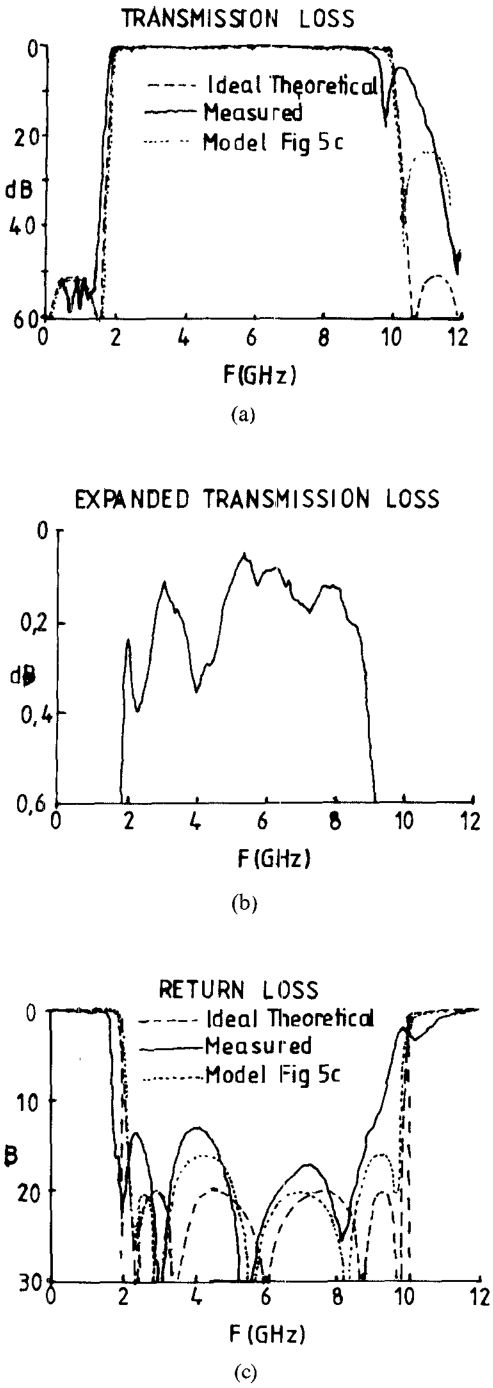

Fig. 7. (a) Transmission response of seventh-order filter. (b) Passband expanded. (c) Return loss.

$f_c = 2$ GHz. The prototype filter is shown in Fig. 5(a), and the high-pass transformed network after frequency and impedance scaling, in Fig. 5(b). The series open circuited stubs are approximated by the short coupled line sections described above, and the shunt resonators as a cascade of two unit elements, using the transform described by Minnis [7]. The final element values are shown in Fig. 5(c).

The artwork for the etching of the filter is shown in Fig. 6. Because of the low impedance of the second unit element in the resonators, these unit elements were realized in microstrip rather than suspended stripline, which was used in the rest of the filter. This microstrip line implementation was only necessary because of the extremely wide design bandwidth. With narrower bandwidths these impedances also become realizable in suspended stripline. The transmission response of the filter is given in Fig. 7(a), as compared with the ideal theoretical response. The variation of return loss versus frequency is shown in Fig. 7(c). The agreement in performance is excellent well beyond the center frequency of the filter. No optimization or subsequent tuning was done on the filter after assembly.

The difference between the theoretical response and the measured results can be partly attributed to manufacturing tolerances, but the main discrepancy at the high frequency end is due to T junction effects which were not included in the model of Fig. 5(c).

VII. Conclusions

A procedure for the design of inhomogeneous coupled line sections has been presented. The approach is to equate the performance of the coupled line section to that of a series stub at two frequencies, thereby ensuring very good performance over a wide portion of the band. The test samples of coupled lines performed very closely to what was theoretically expected.

A trial filter was constructed using the coupled line design procedure; this filter performed extremely well, giving a useful passband to almost the upper cutoff frequency. The power of the design procedure is illustrated by the fact that the filter performance obtained was without any optimization of element values or tuning whatsoever.

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